

Analysis of Students' Errors in Solving Multivariable Calculus Optimization Problems Using Newman's Error Analysis

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Abstract

This study aims to describe the types of errors made by students when solving optimization problems in the Multivariable Calculus course based on Newman Error Analysis (NEA). The study employs a qualitative descriptive approach with simple quantitative analysis in the form of percentages to illustrate the frequency of errors. The research subjects were 34 students in the Mathematics Education program who had taken the Multivariable Calculus course. Data were collected through essay tests and semi-structured interviews. Data analysis was conducted using the Miles and Huberman model, while error classification was based on Newman's five stages: reading, comprehension, transformation, process skills, and encoding. This study identified three types of errors made by students in multivariable calculus optimization problems: (1) transformation errors (58,83%), characterized by an inability to construct a mathematical model and relate the objective function to the constraints; (2) process skills errors (26,47%), characterized by errors in differentiation, algebraic manipulation, and incomplete procedures; and (3) encoding errors (14,70%), characterized by a failure to communicate the final answer. These findings indicate that students' difficulties lie not only in conceptual and procedural skills, but also in mathematical communication. These results can serve as a basis for instructors in designing more effective instruction and diagnostic assessments.

Keywords: Newman Error Analysis, Multivariable Calculus, Error Analysis, Mathematics Education.

Abstrak

Penelitian ini bertujuan untuk mendeskripsikan jenis-jenis kesalahan mahasiswa dalam menyelesaikan masalah optimasi pada mata kuliah Kalkulus Multivariabel berdasarkan Newman Error Analysis (NEA). Penelitian menggunakan pendekatan deskriptif kualitatif dengan analisis kuantitatif sederhana berupa persentase untuk menggambarkan frekuensi kesalahan. Subjek penelitian adalah 34 mahasiswa Program Studi Pendidikan Matematika yang telah menempuh mata kuliah Kalkulus Multivariabel. Data dikumpulkan melalui tes uraian dan wawancara semi-terstruktur. Analisis data dilakukan menggunakan model Miles dan Huberman, sedangkan klasifikasi kesalahan mengacu pada lima tahapan Newman, yaitu reading, comprehension, transformation, process skills, dan encoding. Hasil penelitian menunjukkan bahwa tidak ditemukan kesalahan pada tahap reading dan comprehension. Penelitian ini mengidentifikasi tiga jenis kesalahan mahasiswa pada materi optimasi Kalkulus Multivariabel: (1) transformation error (58,83%) berupa ketidakmampuan membangun model matematika dan menghubungkan fungsi objektif-kendala; (2) process skills error (26,47%) berupa kesalahan diferensiasi, manipulasi aljabar, serta prosedur yang tidak lengkap; dan (3) encoding error (14,70%) berupa kegagalan mengomunikasikan jawaban akhir. Temuan ini menunjukkan bahwa hambatan mahasiswa tidak hanya pada kemampuan konseptual dan prosedural, tetapi juga komunikasi matematis. Hasil ini dapat menjadi dasar bagi dosen dalam merancang pembelajaran dan asesmen diagnostik yang lebih efektif.

Kata Kunci: Newman Error Analysis, Kalkulus Multivariabel, Analisis Kesalahan, Pendidikan Matematika.

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INTRODUCTION

Multivariable calculus is one of the fundamental courses in the mathematics education program because it plays a role in developing analytical thinking, mathematical reasoning, and problem-solving skills (Kashefi et al., 2012). Multivariable calculus occupies a crucial position, serving as an important

bridge that extends the concepts of single-variable calculus into higher-dimensional spaces (Stewart, 2007). One of the most complex topics in this course is optimization problems, specifically, the optimization methods used to determine the maximum or minimum value of a function while taking constraints into account (T. Mkhathshwa, 2023).. Mastering the ability to solve these optimization problems is not only important as part of students' academic competencies but also serves as a foundation in various applied fields such as engineering, economics, physics, computer science, and operations research. Therefore, students are not only expected to understand the underlying mathematical concepts, but also to be able to model contextual problems and apply the appropriate solution procedures.

Despite its importance, solving optimization problems is considered one of the most challenging topics for university students. Unlike single-variable calculus, optimization in higher dimensions requires students to visualize abstract geometric surfaces in R -space, solve complex algebraic systems, and interpret solutions within the context of the problem. In this case, students' cognitive load increases significantly when moving from purely algebraic problems to contextual problems, such as word problems or real-world applications. The complexity of these stages can lead students to make various types of errors, both in the problem-understanding stage and in the mathematical solution stage Shimizu & Kang, 2025). T. P. Mkhathshwa (2022) stated that most students are able to use the Lagrange multiplier method or the substitution method to determine critical points in multivariable optimization problems. However, the majority of students have difficulty verifying that the obtained critical points are truly the optimal solution, and almost half of them have difficulty solving the nonlinear equation systems that arise during the solution process. Thus, lecturers need to intervene to overcome students' learning difficulties and maximize multivariable calculus learning in the classroom, especially on the topic of optimization problems.

Instructors cannot determine the appropriate intervention by looking solely at students' final answers. Instead, an in-depth analysis of the errors students makes when solving contextual problems related to optimization is necessary. In this context, error analysis is a diagnostic approach used to identify the types, causes, and patterns of students' errors during the problem-solving process, thereby providing insight into students' conceptual and procedural understanding (Rushton, 2018). Through error analysis, instructors not only learn that students have arrived at incorrect answers but can also identify the specific steps in the problem-solving process that led to those errors (Cahyaningtyas et al., 2021; Hasanah & Yulianti, 2020). Error analysis serves as a powerful diagnostic tool for understanding students' thought processes and determining at which cognitive stage their reasoning begins to deviate. This information is crucial as a basis for designing instruction (Shimizu & Kang, 2025), providing feedback, and developing remediation activities tailored to students' needs.

One of the most widely used frameworks for error analysis in mathematics education research is Newman Error Analysis (NEA), introduced by Newman (1977). Newman (1977) argued that when solving a mathematical problem, students must go through five stages: reading, comprehension,

transformation, process skills, and encoding. The main strength of Newman Error Analysis lies in its ability to systematically identify the stages of problem-solving where errors occur (Kartono & Suciawati, 2025), ranging from reading information, understanding the meaning of the problem, transforming the problem into a mathematical model, performing the solution procedure, to presenting the final answer. Thus, NEA not only reveals the types of errors students make but also helps explain the sources of these errors, thereby serving as a foundation for designing more appropriate learning interventions.

Various studies have applied Newman Error Analysis to analyze students' errors in various mathematics topics, such as algebra (Cahyaningtyas et al., 2021; Saputra & Cesaria, 2023), linear programming (Oktafia et al., 2020), trigonometry (Nur Ega Septiandi et al., 2020; Wardhani & Argaswari, 2022), quadratic functions (Faradilla et al., 2019; Ly & Baba, 2023), probability (Mahatri et al., 2023), and integrals (Jay & Angco, 2021; Ningsi et al., 2022; Rasul & Citro Trisakti, 2022; Simamora et al., 2024). The results of these studies indicate that transformation errors and process skills errors are the most frequently encountered types of errors. However, research specifically examining students' errors in solving multivariable calculus optimization problems at the undergraduate level remains very limited. This situation highlights a research gap that warrants further investigation, given that solving optimization problems requires students to integrate the ability to model contextual problems, establish mathematical relationships between variables, and apply optimization procedures systematically.

Based on the above discussion, this study aims to describe the types of errors made by undergraduate students in solving optimization problems through the lens of Newman's procedure. By mapping the frequency and systemic nature of these errors, the results of this study are expected to provide a more comprehensive picture of the characteristics of student errors, thereby serving as a basis for instructors to refine teaching strategies, bridge conceptual gaps, and integrate appropriate visualization technologies to minimize students' learning barriers in advanced calculus. Furthermore, this study can enrich the literature on the application of Newman Error Analysis in higher education.

METHOD

This is a qualitative descriptive study aimed at describing the types of student errors in solving optimization problems based on Newman Error Analysis (NEA). Simple quantitative analysis, using percentages, was used to describe the frequency and distribution of errors. The subjects were 34 third-semester students in the Mathematics Education Study Program at the University of Riau, who were taking Multivariable Calculus in the odd semester of the 2025/2026 academic year. All students took an essay test, and five students were selected as interview informants through purposive sampling based on the variety of error types found to represent different error characteristics.

The research instruments consisted of an essay test and a semi-structured interview guide. The test consisted of a single, non-routine essay question designed to measure the ability to solve

optimization problems through mathematical modeling, formulation of objective functions and constraints, application of solution methods, and interpretation of results, thus allowing for various types of errors according to Newman Error Analysis. Interviews were conducted to explore the causes of student errors.

Data validity was achieved through technical triangulation (Denzin & Lincoln, 2018) by comparing the results of the test and interview analysis. Data analysis followed the Miles and Huberman (2019) model, namely data reduction, data presentation, and conclusion drawing. In the reduction stage, student answers were classified based on Newman's five types of errors, then presented in tables and descriptions before conclusions were drawn regarding the dominant errors and their causes.

Error analysis refers to Newman's five stages of Error Analysis: reading, comprehension, transformation, process skills, and coding (Newman, 1977). Each student was classified based on the primary error, which is the first error to appear in Newman's sequence of stages, as errors in one stage can influence subsequent stages. Subsequent errors were analyzed qualitatively as consequences of the primary error. Incomplete answers were also analyzed based on the last stage successfully completed and confirmed through interviews. Thus, error classification was based on both written answers and students' thinking processes. Indicators for each Newman stage are presented in Table 1.

Table 1. Newman's Stage Error Indicators

Newman's Stages	Error Indicators
Reading	Students misread symbols, notation, key terms, and information in the problem.
Comprehension	Students fail to identify known and unknown information in the problem.
Transformation	Students fail to transform the information in the problem into a mathematical model (objective function, constraint functions, and Lagrange function).
Process Skills	Students make errors in performing solution procedures, such as partial differentiation, algebraic manipulation, and solving systems of equations.
Encoding	Students fail to write down their conclusions, make errors in writing units, or their conclusions do not address the core of the question.

The percentage of student errors at each stage is calculated using the following formula:

$$P = \frac{f}{N} \times 100\%$$

Notes:

P = Error percentage

f = Frequency of errors at a specific stage

N = Total number of errors made by all students

RESULTS AND DISCUSSION

Based on the analysis of the answer sheets, it was found that students used various strategies to solve optimization problems. Some students chose the Lagrange multiplier method, while others used the substitution method by converting the constraint function into the objective function. Although the strategies used differed, both approaches can yield correct solutions when applied correctly. Table 2 below summarizes the errors made by the students.

Table 2. Summary of Students Error

Newman's Stage	Percentage (%)	Error Description
Reading	0	No errors found at this stage
Comprehension	0	No errors found at this stage
Transformation	58.83	<ul style="list-style-type: none"> • Failed to convert the word problem into a mathematical model • Errors in determining the objective function and constraint functions
Process Skills	26.47	<ul style="list-style-type: none"> • Differentiation errors • Algebraic manipulation errors • Procedures not completed thoroughly
Encoding	14.70	<ul style="list-style-type: none"> • Failure to include appropriate units • Incomplete answers • Failure to write a final conclusion consistent with the context of the problem

Based on Table 2, it was found that the most prevalent errors occurred during the transformation stage, accounting for 58.83%. The next stage was process skills, with an error rate of 26.47%. The encoding stage accounted for 14.70%, and no errors were found in either the reading or comprehension stages—both at 0%. This indicates that all participants were able to read the questions correctly and understand the information presented and asked in the given problems. Meanwhile, during the transformation stage, students struggled to convert the problems into mathematical models. The most common type of error was a failure to fully define variables and formulate the objective function for surface area. Additionally, some students made mistakes in determining the objective function or constraint functions, resulting in mathematical models that did not accurately represent the problem.

Errors at the process skills stage include mistakes in the differentiation process, algebraic manipulation errors, and failure to complete the solution procedure to obtain the final result. This indicates that although students have been able to formulate mathematical models, some still face difficulties in performing calculations accurately and systematically. Furthermore, errors that appeared at the encoding stage included failing to include the appropriate units, providing incomplete answers (for example, only determining the size without calculating the minimum area), and failing to write a final conclusion that was appropriate to the context of the problem. These errors indicate that some students are not yet accustomed to presenting final results completely and in accordance with the requirements of the problem.

Transformation Error

A transformation error is an error that occurs when students have understood the information provided and asked in a problem but are not yet able to transform the problem into a mathematical model or determine the appropriate solution strategy. At this stage, students are aware of the context of the problem but have difficulty converting that information into a mathematical representation that can be used to obtain a solution. In this study, transformation errors were demonstrated by students who were unable to construct a mathematical model correctly, thereby preventing them from continuing the problem-solving process. One example of such an error is shown in Figure 1 below

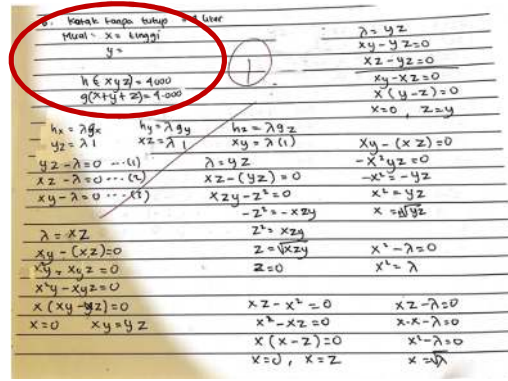


Figure 1. Student S1's Answer

Based on the answer sheet in Figure 1, it shows that the students have defined variables for the length, width, and height of the box and understand that the volume of the box is constant. However, the students were unable to fully define the variables or transform this information into a mathematical model consisting of an objective function and constraint functions that could be used to solve the optimization problem. As a result, the problem-solving process stopped at the variable-assignment stage without producing an appropriate solution strategy. This was corroborated by the interview results with subject S1; Student S1 stated that he did not encounter any problems during the “reading” stage, as evidenced by his ability to identify the key information in the problem, namely, that the box’s capacity is 4 liters. The student also demonstrated an understanding of the problem’s context by stating that the volume of the box is related to the product of its three dimensions, involving three variables (x, y, and z). However, the student admitted to being confused about how to model this problem accurately.

This statement indicates that the students’ main difficulty lies not in the process of differentiation or algebraic manipulation, but rather in the stage of transforming contextual problems into mathematical models. As a result of this error, the students performed differentiation and algebraic manipulation based on an inappropriate model, thus failing to obtain either the optimal measure or the minimum area of the box. Consequently, the procedural errors that arose in the subsequent stages were a consequence of the transformation error that occurred in the initial stage.

Another form of transformation error was demonstrated by students who had successfully formulated the constraint function and part of the mathematical model but were unable to determine the next step in the solution process. Student S2’s answer sheet is shown in Figure 2

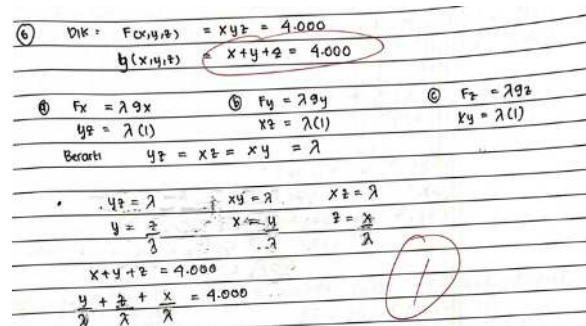


Figure 2. Student S2's Answer

In contrast to S1 students, S2 students were able to identify that the solution involves an objective function and a constraint function. However, the students made a mistake in distinguishing between the two functions. They treated the volume function as the objective function to be optimized, while the constraint function was written as $x + y + z = 4000$. As a result, the entire process from differentiation and algebraic manipulation to determining the solution is carried out based on a mathematical model that does not correspond to the problem. Based on interview results, students stated that they know the formulas for volume and surface area but do not understand how these two concepts are connected in solving optimization problems. The errors made by S2 indicate that the students already possess knowledge of the concepts of volume and surface area but do not yet understand the roles of each concept within the optimization model. This suggests that the errors are conceptual in nature—specifically, a failure to identify which function serves as the objective function and which serves as the constraint function.

Based on Newman's stages, these errors are categorized as transformation errors because the students successfully completed the reading and comprehension stages but failed to identify an appropriate mathematical model or problem-solving strategy. Errors at this stage prevent students from proceeding to the process skills stage, thereby preventing them from carrying out the problem-solving procedure. Furthermore, these findings indicate that transformation errors are not singular in nature but manifest in several distinct characteristics. The first characteristic is exhibited by students who are unable to transform contextual information into a mathematical model. The second characteristic is demonstrated by students who have partially constructed a mathematical model but incorrectly determine the relationship between the objective function and the constraint functions. These distinct characteristics indicate that transformation errors are not only related to the ability to translate language into mathematical symbols but also involve the ability to integrate various mathematical concepts into a complete optimization model.

The findings of this study are consistent with the results of Oktafia et al. (2020), which show that transformation errors in linear programming problems occur when students have difficulty transforming the information in a problem into a mathematical model, particularly in accurately determining the objective function and constraint functions. Similarly, Ekasari & Putra (2024) reported that transformation errors arise because students are not yet able to interpret the problem situation and connect the available information with relevant mathematical equations. These similarities indicate that transformation errors are not only caused by an inability to translate verbal information into mathematical symbols but also by a weak ability to construct mathematical models appropriate to the problem context. Furthermore, the results of this study also support the findings of Ningsi et al. (2022), Rasul & Citro Trisakti (2022) and Wardhani & Argaswari (2022), who state that errors at the transformation stage will lead to errors in subsequent solution stages, in accordance with the hierarchical characteristics of Newman's Error Analysis. This was evident in this study, where students who constructed mathematical models incorrectly subsequently performed differentiation and algebraic

manipulation based on those flawed models, resulting in solutions that did not align with the problem's objectives.

Nevertheless, this study provides more specific findings in the context of multivariable calculus optimization. Unlike previous studies, which generally identified transformation errors as failures to construct mathematical models, this study shows that transformation errors have two distinct characteristics. The first characteristic is exhibited by students who are unable to transform contextual information into a mathematical model at all. The second characteristic is exhibited by students who have successfully constructed part of a mathematical model but are unable to integrate the concepts of volume, surface area, objective function, and constraint function into a complete optimization model. These findings indicate that in the context of multivariable calculus optimization, transformation errors are not only related to the ability to model problems but also to the ability to connect various interrelated mathematical concepts within a single problem-solving process.

Process Skills Error

A process skills error is a mistake that occurs when students have been able to understand the problem and construct an appropriate mathematical model but have difficulty applying the solution procedure to arrive at the correct answer. One example of a process skills error is shown in Figure 3 below.

Handwritten student work on lined paper. The student has written a problem statement in Indonesian: "C) misal Panjangnya = x dm, lebarnya = y dm, tingginya = h dm, x.y.h = 9. tanpa tutup maka F = xy + 2xh + 2yh. h = 9, xy. F = xy + 2x(9/x) + 2y(9/xy). Then they differentiate: f_x = y + 0 + 8/x^2, f_y = x + 8/y. They set these to zero: y + 8/x^2 = 0, x + 8/y = 0. They solve for y: y + 8/x^2 = 0, y = 2. The final answer is y = 2.

Figure 3. Student S3's Answer

Figure 3 shows that among third-year students, process skills errors appear in two forms. First, students make procedural errors in the differentiation process, resulting in incorrect derivatives. For example, students differentiate the function $\frac{8}{x}$ to obtain $\left(+\frac{8}{x^2}\right)$ when the correct derivative is $\left(-\frac{8}{x^2}\right)$. This error caused the resulting equation to be incorrect, preventing the subsequent solution process from being carried out correctly. The second error was that students were unable to solve the system of equations resulting from the differentiation independently and instead substituted the solution process with an answer they recalled from previous experience. On the answer sheet, students directly wrote the value $y = 2$ without showing the solution process supporting that result. Based on the interview

results, the student revealed that he was actually unsure about the calculation process he had performed and determined the value based on memory because he had previously obtained the same result on a similar problem. This finding indicates that the correct final answer is not always obtained through the application of correct mathematical procedures but can stem from recalling a previous solution.

These findings are consistent with the research by Jay & Angco (2021), which shows that process skills errors in solving calculus problems are largely caused by errors in performing symbolic operations, specifically, errors in using positive and negative signs, which result in subsequent solution procedures becoming inaccurate. Similar findings were also reported by Roman (2024), who identified that one of the errors students frequently make in differential calculus is an error in differentiating negative-power expressions. Additionally, Wardhani & Argaswari (2022) stated that a lack of precision and focus during calculations often causes students to make procedural errors even when the chosen solution strategy is appropriate. The results of this study reinforce these findings but also indicate that process skills errors are not limited to computational errors. Based on the interview results, students determined the value of $y = 2$ based on their memory of previously learned problem-solving methods, rather than through a process of mathematical reasoning. Thus, a correct answer does not necessarily reflect mastery of the correct problem-solving procedure.

Another type of process skills error is demonstrated by students' inability to complete the entire problem-solving procedure, as shown in Figure 4.

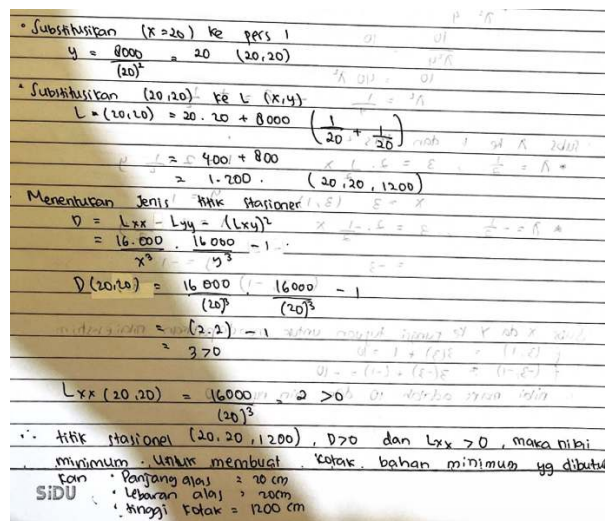


Figure 4. Student S4's Answer

In this case, the student successfully built a mathematical model, determined the stationary point, and obtained a minimum value of 1200 cm for the objective function. However, the student did not continue the procedure by substituting the values of x and y into the constraint function $h = \frac{4000}{xy}$ to determine the height of the box. As a result, the student assumed that the minimum value of the objective function was the height of the box and concluded that the dimensions of the box were $(20, 20, 1200)$. The interview revealed that the student did not recall that a step was still needed to determine the value

of h and believed that the value 1200 was the height of the box. In this case, it can be concluded that the error was not caused by an inability to construct a mathematical model, but rather by an incomplete application of the solution procedure until all variables requested in the problem were obtained.

The findings regarding S4 students also align with the research by Jay & Angco (2021), which states that process skills errors can manifest as “unfinished answers”, that is, students have followed the correct procedure but stopped working on the problem before completing all necessary steps. Similar results were also found by Oktafia et al. (2020), who reported that students often fail to complete the calculation process thoroughly even though the initial steps of the solution were performed correctly. Additionally, Anggraini & Surya (2025) noted that some students struggle to determine the next step in the solution process, causing the procedure to stop before all the problem’s requirements are met. In this study, this characteristic was evident when students successfully obtained the minimum value of the objective function but did not continue the procedure to determine the upper bound of the box through the constraint function, thereby assuming the minimum value of the function to be the upper bound. These findings indicate that process skills errors are not always caused by computational errors but also by an inability to maintain consistency in the solution procedure until all variables required by the problem are obtained.

In general, process skills errors in this study were not limited to mistakes in performing mathematical operations but also included an inability to follow problem-solving procedures systematically and completely. Some students made errors in algebraic manipulation and differentiation, while others were able to perform the initial calculations but failed to proceed to the next steps in the solution, stopped the procedure before all requirements of the problem were met, or relied on their memory of previously learned solutions. These findings align with the research by Ningsi et al. (2022) and Rasul & Citro Trisakti (2022), which showed that students often solve problems by following example solutions without understanding the procedures conceptually. Furthermore, Oktafia et al. (2020) emphasized the importance of encouraging students to practice systematically so they can complete the entire problem-solving procedure, while Simamora et al. (2024) stated that students need guidance to connect concepts with structured problem-solving steps. Thus, the results of this study indicate that mastery of optimization concepts alone is insufficient to produce correct solutions if students are unable to apply the problem-solving procedures completely, consistently, and systematically.

Encoding Error

An encoding error is an error that occurs during the final answer presentation stage—that is, when students have successfully completed the entire problem-solving process but are unable to communicate their results in accordance with the question’s requirements. At this stage, students have actually arrived at the correct answer, but their results are not presented completely or are not stated in a way that directly answers the question posed. In this study, encoding errors were demonstrated by students who had successfully completed all stages of problem-solving. The students’ answer sheets are shown in Figure

5.

The image shows a student's handwritten work on lined paper. The work is as follows:
- $a = \sqrt[3]{8000}$
- $a = 20 \text{ cm.}$
- $\text{Maka ukuran alas} = 20 \times 20$
-
$$h = \frac{V}{xy} = \frac{4000}{20 \times 20}$$

-
$$= \frac{4000}{400} = 10 \text{ cm.}$$

- Luas minimum:
-
$$A_{\min} = xy + 2V \left(\frac{1}{x} + \frac{1}{y} \right)$$

-
$$= 20(20) + 8000 \left(\frac{1}{20} + \frac{1}{20} \right)$$

-
$$= 400 + \frac{8000 \cdot 20}{20}$$

-
$$= 400 + 800$$

-
$$= 1.200 \text{ cm}^2.$$

Figure 5. Student S5's Answer

Based on the answer sheet in Figure 5, all steps of the solution were performed correctly. The students successfully formulated the objective function and constraint functions, determined the optimal point, and calculated the minimum material area according to the procedure. The results show that the optimal dimensions of the box are 20 cm in length, 20 cm in width, and 10 cm in height, with a minimum material area of 1200 cm². This indicates that the student made no errors in the reading, comprehension, transformation, or process skills stages. However, the student did not present the final answer in the form of a conclusion that explicitly answered the question posed in the problem. Their answers stopped at the calculation results without a statement integrating those results into the context of the problem. Consequently, the reader must conclude on their own that the optimal dimensions of the box are 20 cm in length, 20 cm in width, and 10 cm in height, with a minimum surface area of 1200 cm².

The interview results reinforce these findings. The student explained that he felt the problem-solving process he had carried out was sufficient, so he did not write a final conclusion. The student believed that the calculations written on the answer sheet already represented the answer to the given question. This statement indicates that the error was not caused by an inability to understand the concepts or apply the problem-solving procedures, but rather by a lack of attention to presenting a final answer that met the requirements of the question.

The findings of this study are consistent with those of Ashiddiqi & Nurul hidayah (2026) who state that encoding errors occur when students fail to write down conclusions or final answers based on the results they have obtained. Similar results were also reported by Ly & Baba (2023), who showed that students can still experience encoding errors even if the entire problem-solving process was carried out correctly, because they failed to communicate the final solution in a form that met the requirements of the problem. This similarity indicates that the ability to obtain correct calculation results is not always accompanied by the ability to explicitly present the final answer. The interview results in this study also reinforce the findings of Ashiddiqi & Nurul hidayah (2026), in which students considered the calculation results they had obtained sufficient to represent the answer, so they no longer formulated a

final conclusion. Furthermore, Seng (2020) states that students' lack of practice in writing conclusions, coupled with a lack of guidance from instructors, is one of the causes of encoding errors. Therefore, mathematics instruction needs to accustom students not only to obtaining correct calculation results but also to interpreting and communicating those results in accordance with the context of the problem.

Nevertheless, this study reveals encoding error characteristics that differ from those found in most previous studies. In many studies, encoding errors generally arise as a consequence of errors in the transformation or process skills stages, resulting in an incorrect final answer. In contrast, in this study, encoding errors occurred independently (pure encoding errors)—that is, when students successfully completed the reading, comprehension, transformation, and process skills stages correctly but still failed to provide a final answer that met the question's requirements. This finding suggests that encoding errors are not always the result of procedural mistakes but may also reflect weaknesses in mathematical communication skills, particularly in integrating calculation results into a conclusion that explicitly addresses the problem. Therefore, mathematics education must not only develop conceptual and procedural skills but also accustom students to communicating their solutions comprehensively, contextually, and in accordance with the given questions.

CONCLUSION

This study aims to describe the types of student errors in solving optimization problems in Multivariable Calculus courses based on Newman Error Analysis (NEA). The results indicate that errors only occur at the transformation, process skills, and coding stages, while reading and comprehension errors are not found. This indicates that students generally understand the information and questions, but still experience difficulties in modeling the problem, applying the solution procedure, and communicating the results.

At the transformation stage, errors involve the inability to transform contextual information into a mathematical model and integrate the objective function and constraints into the optimization model. At the process skills stage, errors include differentiation, algebraic manipulation, incomplete completion of procedures, termination of the process before all conditions are met, and reliance on memory of previous solutions. Meanwhile, coding errors occur when students obtain the correct solution but fail to present a conclusion that meets the requirements of the problem.

This study extends the application of Newman Error Analysis to the context of multivariable calculus optimization by demonstrating that transformation, process skills, and coding errors have more diverse characteristics than previously found. Process skills errors include not only computational errors but also premature termination of procedures and reliance on memory. Furthermore, pure coding errors were found, namely the failure to communicate the solution even though the entire solution process had been carried out correctly. This finding suggests that good procedural skills do not necessarily translate to good mathematical communication skills.

The implication is that optimization learning in Multivariable Calculus needs to emphasize the

ability to build mathematical models, systematically complete procedures until complete, and communicate results through contextually appropriate conclusions. The results of this study can also serve as a basis for lecturers in designing learning strategies, diagnostic assessments, and remediation that are appropriate to the characteristics of student errors.

This study has several limitations. First, the research subjects were only from one Mathematics Education Program class at one university, so the results cannot be generalized. Second, the instrument used only one optimization problem, so the variation in errors was limited. Third, the analysis only used Newman Error Analysis, so other aspects, such as cognitive load, representation, and mathematical communication, were not examined in depth. Therefore, future research is recommended to involve more subjects, use a wider variety of optimization problems, and combine Newman Error Analysis with other analytical frameworks to obtain a more comprehensive picture of students' thought processes in solving optimization problems.

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